Estimating the Gravitational Wave Energy, Fractional Mass Loss, and Luminosity Distance of BBH Merger Events in the LIGO O3-O4 Data Catalogs

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Abstract

In this work, we present a Bayesian analysis of binary black hole (BBH) merger events of the Laser Interferometer Gravitational-Wave Observatory's (LIGO's) third observing run (O3) and the newer O4 catalogs, using computational tools both from the Gravitational Wave Open Science Center (GWOSC) and models developed by our group. We utilize open-source computational tools to simulate previously observed BBH events and pull the resulting data for our analysis. We first find a ~4% mass-energy loss to gravitational wave emission from several dozen of our simulated events, which is consistent with theoretical predictions. We then examine how an event's total mass, duration, peak frequency, and signal-to-noise ratio relate to each other and how they can be used to infer the event's luminosity distance and create an independent determination of the mass-energy loss. We compare these findings with an analysis carried out by our previous physics-inspired model for predicting luminosity distances (cf. Celebration of Scholars 2024). Finally, to improve the accuracy and physical understanding of our results, we apply Bayesian parameter estimation to construct confidence intervals around the mass-energy loss.

Introduction and Background

- Gravitational waves were first predicted by Einstein's theory of General Relativity in 1916
- These "ripples" in spacetime are commonly caused by merging binary systems (black hole and neutron star pairs)
- First detection of gravitational waves was made by LIGO in 2015
- LIGO operates between observing runs
  - O3 denoted its third observing run (lasting from April 2019 - March 2020)
  - O4 denotes the current observing run (which began in May of 2023)
- O3 identified 90 events by a global network of detectors and O4 has detected 84 events to date

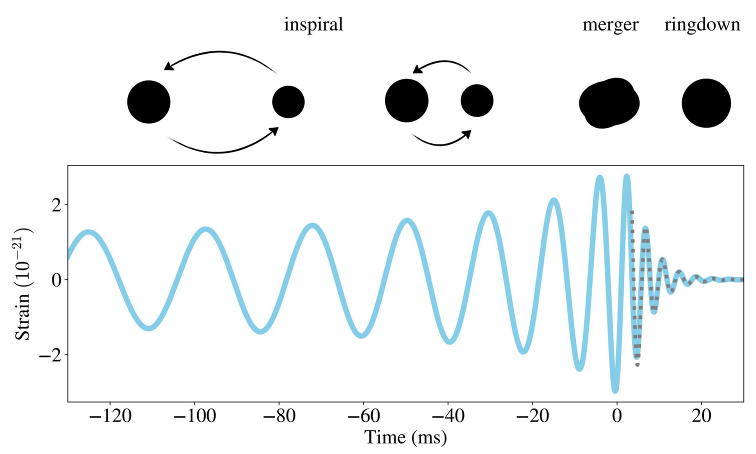

Figure 1: Expected waveform from a binary inspiral broken into its three phases [1]

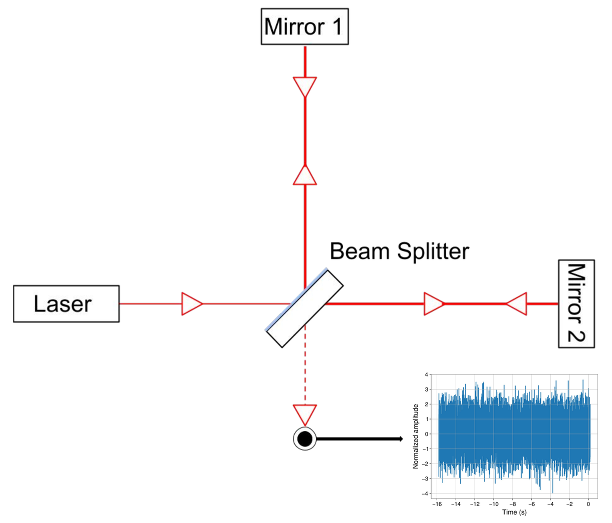

Figure 2: Basic LIGO schematic and its signal

Bayesian Analysis

- Method of statistical inference that allows one to update their knowledge of parameters upon seeing data
- Result, called the **posterior** distribution, gives a range of probable values for the parameters of interest

$$\underbrace{P(\Theta|D)}_{\text{Posterior}} = \frac{\overbrace{P(\Theta)}^{\text{Prior}}\overbrace{P(D|\Theta)}^{\text{Likelihood}}}{\underbrace{P(D)}_{\text{Evidence}}}$$

- **Prior** encodes our knowledge of parameters *prior* to seeing data
- **Likelihood** is how likely the observed data is to be generated by the parameters

Pre and Post Merger Parameter Estimation of BBH Merger Events

1. Take time of event, t_{GPS} from `GWOSC` package [1]
2. Pull *live data* from specified interferometer near t_{GPS}
3. Define prior distributions $P(\Theta_i)$ for parameters values of interest $\Theta_i \in \{\mathcal{M}, D_L, \chi, \ldots\}$
4. Use waveform generation tool `IMRPhenomXP` to construct model M_i in likelihood $\mathcal{L}(\Theta_i) \propto \exp\left(-\sum_t \frac{(D_t - M_t)^2}{2\sigma_t^2}\right)$
5. Run `Bilby` sampler on $P(\Theta_i)\mathcal{L}(\Theta_i)$ to construct posteriors $P(\Theta_i|D)$

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6. Use physics-based `LALSimulation` tool to compute M_f, given Θ_i, for every point in $P(\Theta_i|D)$
7. Calculate $M_{\%}$ using $\mathcal{M}$ and M_f

- The analysis detailed above yields posterior distributions for the parameters of interest, of both the initial component black holes and the final merger black hole.
- The posteriors of our parameters of interest, namely the initial mass of both black holes, the final merger mass, and the percentage of mass lost to GW emission, are plotted in a corner plot. An example of such a plot is given to the right for the event GW170729

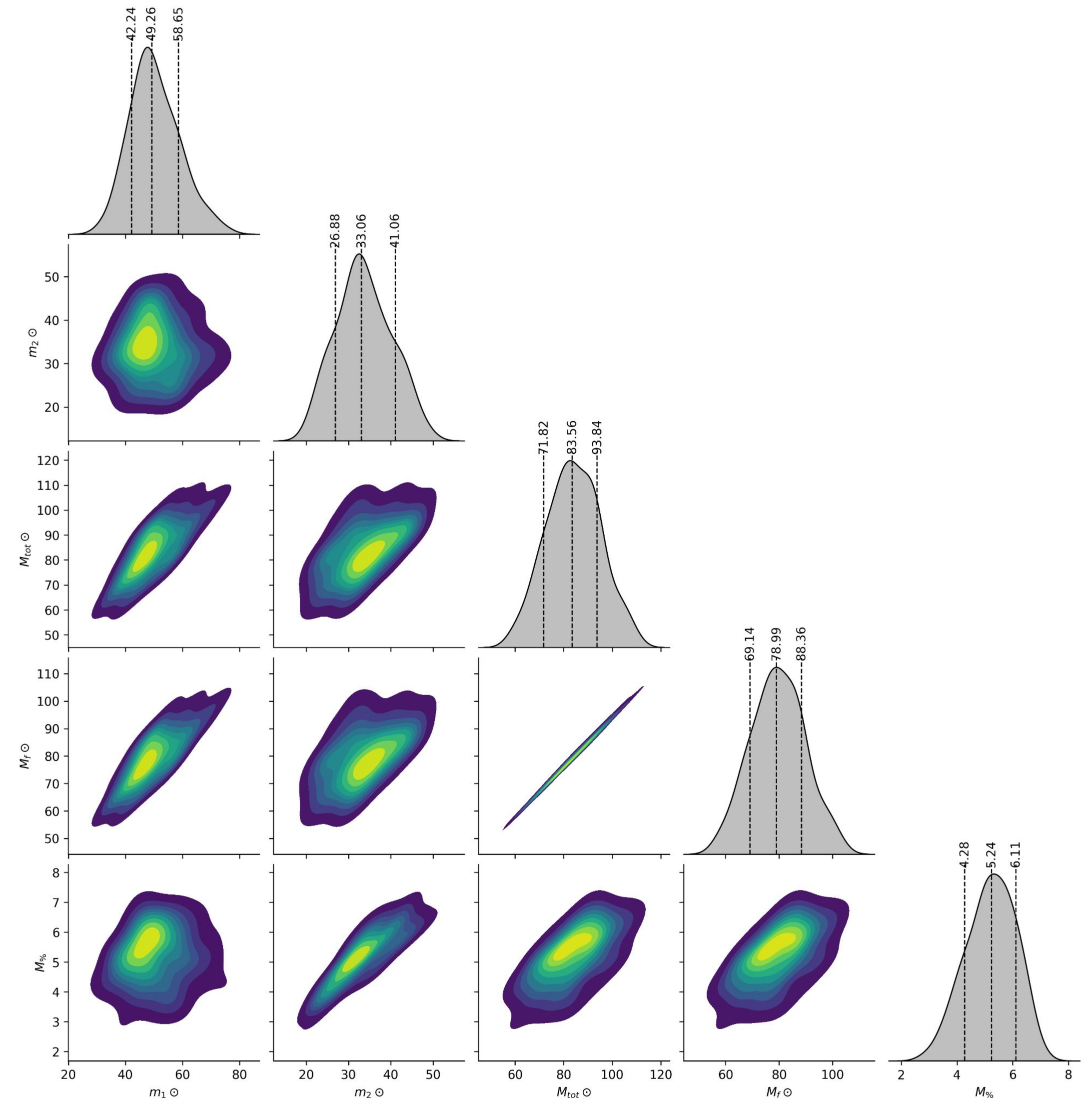

Figure 3: Mass results from Bayesian analysis and LALSimulation displayed in a corner plot for GW170729

Mass Loss Distribution for All Events

- The energy released in gravitational waves by a merger event is approximately the difference between the final and initial mass of system:

$$E_{GW} = \Delta M C^2$$

- For each event's posterior, we take the median $M_{\%}$ and histogram these values
- Upon histogramming these values, we find a BBH merger has a typical mass loss of 4.83%

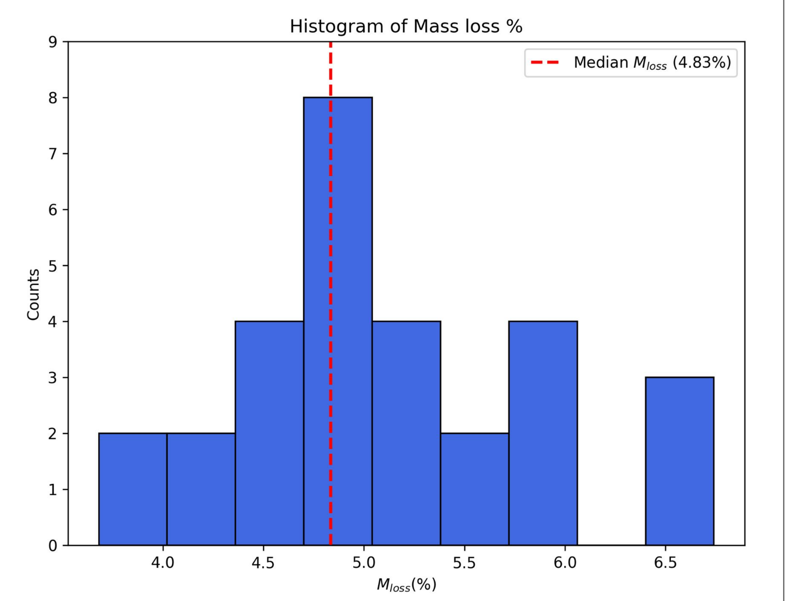

Figure 4: Distribution of mass loss percentage across all events

Mass Loss Estimation Through Numerical Integration

- The orbital frequency ω over time of two orbiting black holes can be calculated using Keplerian dynamics with the total mass M and reduced mass μ [2]:

$$\dot{\omega}^3 = \left(\frac{96}{5}\right)^3 \frac{\omega^{11}}{c^{15}} G^5 \mu^3 M^2$$

- Assuming a final orbital frequency of 75 Hz, we solve for the orbital frequency of the BBHs over time using numerical integration.
- General relativity tells us the luminosity of emitted gravitational waves during infall is a function of the orbital frequency ω, orbital radius r, and reduced mass μ [2]:

$$\frac{\mathrm{d}}{\mathrm{d}t} E_{\mathrm{GW}} = \frac{32}{5}\frac{G}{c^5}\mu^2 r^4 \omega^6$$

- By comparing these values to the total mass energy of the system we find the mass loss fraction to be ~3.3%

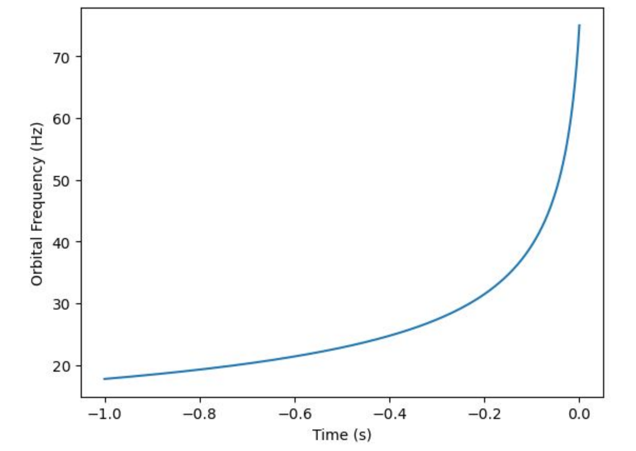

Figure 5: Result of numerical integration of the orbital frequency with respect to time for a sample event.

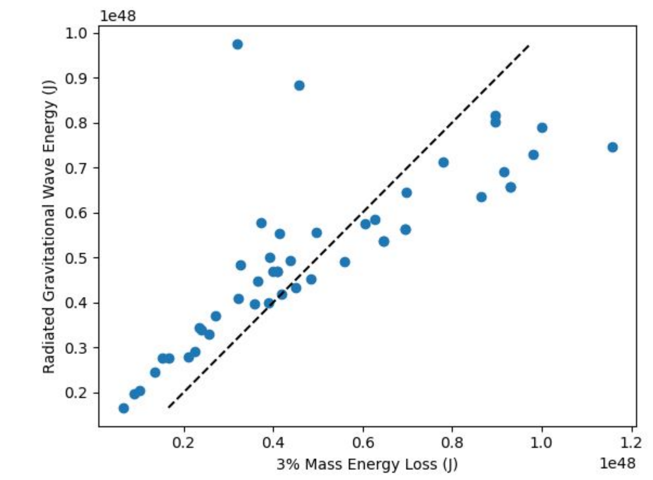

Figure 6: Fitting the radiated gravitational wave energies found through numerical integration to the mass energy yields a mass loss faction of ~3.3%

Revised Luminosity Distance Model

- Comparing the luminosity emitted at peak frequency to the luminosity observed by LIGO at peak strain, we compute the luminosity distance of an event using the expression:

$$d_L = \frac{8G^{5/3}}{c^4}\sqrt{\frac{2}{5}}\frac{\mu(M\omega)^{2/3}}{\mathrm{SNR}\cdot N_{\mathrm{thresh}}\cdot h_{\mathrm{strain}}}$$

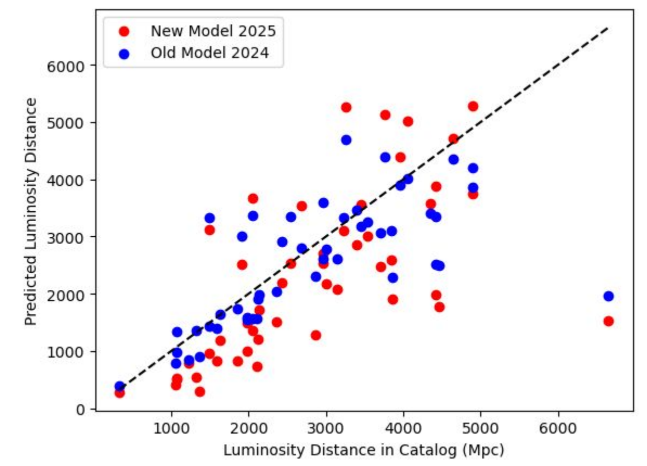

Figure 7: Comparing this work's model to luminosity distance estimator model we introduced in 2024 [3]. It is shown these methods yield comparable predictions.

Acknowledgements

[1] R. Abbott et al. (LIGO Scientific Collaboration, Virgo Collaboration and KAGRA Collaboration), "Open data from the third observing run of LIGO, Virgo, KAGRA and GEO", ApJS 267 29 (2023)

[2] LIGO Scientific and Virgo Collaborations, et al. "The Basic Physics of the Binary Black Hole Merger GW150914." *Annalen der Physik* 529.1-2 (2017): 1600209.

[3] Lucas Peterson, Andrew Valentini, Kaitlyn Prokup, Hiroki Imura. "Estimating the Luminosity Distance and Mass Properties of BBH Merger Events in LIGO O4 Data." (2024)